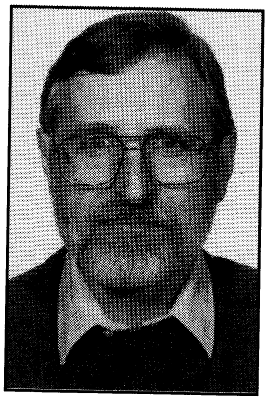
The Role of Manipulatives in Mathematics Education

By Thomas O'Shea



Thomas O'Shea

One of the fondest desires of the professional mathematics education community is that teachers will adopt the use of manipulative materials to teach mathematics at all levels. The use of manipulative devices is being extensively promoted in the professional literature of the National Council of Teachers of Mathematics (e.g., NCTM, 1989; NCTM, 1991). The current effort to convince teachers to use such materials is only the latest in a long series of similar appeals. A schism developed between how mathematics is taught and how it is learned, and the use of manipulatives can help bridge this gap.

The seven liberal arts that formed the basis for education up to the late Renaissance period contained four areas of study related to mathematics: arithmetic, geometry, music, and astronomy. These, in turn, were connected to the more fundamental notions of number and space. Arithmetic was thought of as the study of numbers at rest, and geometry as the study of figures (in the sense of geometric shapes) at rest. Music, on the other hand, consisted of the study of harmonics originally based on the Pythagorean analysis of vibrating strings, and was viewed as the study of numbers in motion. Astronomy, centered on the Ptolemaic theory which assumed the circular motion of planets, was the study of figures in motion.

In all four areas, mathematics was centered on or closely related to physical devices. The abacus was used to carry out numerical computations. Euclidean geometry was based solely on constructions that were possible with the straight edge and compass. Music was an experimental activity based on vibrating strings. Astronomy relied on the use of observational instruments such as the astrolabe. Those who practiced in these areas were expected to be familiar with the devices as well as the mathematics that governed their use.

In the specific area of arithmetic, the Hindu-Arabic system of numerical notation was adopted gradually in Europe over a period of 400 years, roughly from 1200 to 1600. As a consequence, the basis of computing underwent a radical change. Initially, one used the abacus or counting board to carry out calculations and recorded the results using Roman numerals. By the end of the period, the standard practice was to operate solely on numerical symbols on paper. The skills of the *abacist* were thus replaced by those of the *algorist*, or one capable of applying the appropriate algorithms. This move was a welcome relief from the tedium and complexity of carrying out the operation of multiplication and the more complex operation of division on the abacus. On the other hand, such a shift replaced the manipulation of devices with the manipulation of symbols, with the concomitant danger that procedures could be simply memorized without understanding the underlying rationale of the algorithm.

By the end of the 1600s, probably fewer than 400 men in all of England could be said to be competent in mathematics and its applications (Cohen, 1982, p. 89). The vast majority of the population learned only enough arithmetic to find the correct verse in the Bible or knew the basics of commercial arithmetic as a result of self-study.

The few schools in existence at that time focused on arithmetic theory and Euclidean proof and failed to prepare people adequately to deal with changes in society and trade. The mathematical practitioners of the 17th and 18th centuries arose to fill the void, and one can see the extent to which practical matters were important in the advertisement for one William Alingham, mathematical practitioner (fl. 1694-1710): "At the house of Lord Weymouth's in Chanel Row, Westminster, [learners] are taught

THOMAS O'SHEA is an associate professor in the faculty of Education at Simon Fraser University. His prime area of interest is mathematics education and the role of manipulatives and application in teaching mathematics at all levels.

arithmetic, geometry, trigonometry, navigation, surveying, measuring, fortification, and throwing of bombs, with several parts of mathematics" (Taylor, 1954, p. 289).

With the rise of quantification in the 18th century, and the recognition of the need for basic standards of education, more formal school systems were developed. The models for such schools were drawn from the existing Latin grammar schools and academies which focused on abstract and theoretical mathematics including Pythagorean number mysticism. The educational structures developed by the mathematical practitioners were not adopted, and instruction in arithmetic became formalized, theoretical, and divorced from practice. The teacher dictated rules and examples of algorithmic procedures; students simply entered these into their copy books and then completed similar problems using the algorithm.

The use of manipulative devices is being extensively promoted in the professional literature . . .

Reformers such as Pestalozzi (1746–1827) argued for a different approach. In his theory of organic development, he argued that the intellect develops in terms of the experience furnished through sense impressions. The child must experience objects, hence the term "object lessons." Learning must move from the concrete to the abstract. In such ways, arithmetic, previously a subject suitable only for older students, could be understood by children in the first grade. In the United States, Samuel Goodrich in 1818 applied Pestalozzi's ideas to the teaching of arithmetic, and argued that children should discover rules by manipulating tangible objects such as counters and bead-frames (Cohen, 1982, p. 134). This led to the concept of "mental arithmetic" and inductive reasoning in mathematics which was very successfully promoted by Warren Colburn in 1821. Colburn, however, claimed that "the mind is also greatly assisted in the operations by *reference* [added italics] to sensible objects" (Bidwell & Clason, 1970, p. 16). Thus, his advice for teaching arithmetic centered on verbal examples related to practical matters rather than having the children use the "sensible objects" themselves.

Later "methods" books for teachers of arithmetic promoted the use of materials. Wentworth and Reed's 1885 text suggested that the teaching of number facts and elementary work in fractions should be "objective" (Bidwell & Clason, 1970, p. 99). Grube's 1888 text advocated that the "elementary teaching of number should proceed from observation, or, better it should proceed from things" (Bidwell & Clason, 1970, p. 107), although it is clear that it was the teacher who was to do the manipulating, and the student was to do the observing. Speer's 1897 text for teachers was based on the writings of Herbert Spencer who made comparisons of basic objects to thought. Students were to come to understand ratio by making comparisons through seeing, hearing, touching, building, finding, cutting, and drawing (Bidwell & Clason, 1970, p. 168).

During the 20th century, various educational theorists took positions that focused on the child as learner rather than as recipient of teaching. Dewey (1938, p. 45) argued that the problem with traditional education was not that educators did not provide suitable environments, but that they neglected to take into account the other factor important in experience, namely the child's powers and purposes. Piaget (1971) suggested that children form concepts through reconstructing reality, not through imitating it, and he emphasized the centrality of concrete experience for young learners. Bruner (1960) proposed a model in which learners move through three stages of learning. The first, or "enactive" phase, required hands-on or direct experience.

Within the mathematics education community, various commissions have advocated the use of materials to teach mathematics. The 1911 report of the International Commission on the Teaching of Mathematics in the Elementary Schools of the United States (Bidwell & Clason, 1970, 296-298) contains a section on "objective teaching" that might have been abstracted from present-day documents. It called for a broadening of the notion of concrete experience to include "quasi-objective" materials such as pictures, and noted the correlation between the use of materials, inductive teaching, and student "self-activity." In 1919, a major report of the Mathematics Association in England advocated setting up mathematics laboratories in secondary schools where students might determine the mathematics underlying natural phenomena (Mathematics Association, 1928, p. 16). The Mathematics Association itself had been formed in 1871 (under a different name) by a group of teachers who wished to break away from the tradition of teaching geometry solely through the study of Euclid's "Elements." It is ironic that by the 1950s, the Mathematical Association had become so conservative that it, in turn, spawned a subgroup under the leadership of Caleb Gattegno which formed a rival organization known as the Association for Teaching Aids in Mathematics as "in part, a reaction against the sterile and 'academic' treatment of mathematics in so many classrooms" (Cooper, 1985, p. 70). The aim of the Association was to improve mathematics pedagogy particularly by engaging students in mathematical discovery through the increased use of concrete materials.

Prominent individual mathematics educators have also promoted child activity and the use of manipulatives. In the 1953 NCTM Yearbook, Van Engen concluded, on the basis of theory and experimental work of the time, that "perceptual manipulatory activities are of utmost importance in the development of number concepts as well as concepts in general" (p. 91). The same Yearbook devoted a complete chapter to examining the relationship between sensory learning and mathematics and included 47 examples of appropriate activities (Syer, 1953). In 1963, Dienes suggested that "Mathematics is the study of structures, [and] it is painfully obvious that such structures must in the last resort be built up out of our experiences as we cope with our environment" (p. 115). Even during the period that we tend to think promoted a highly abstract system known as the "new mathematics," Fitzgerald and Vance (1970) argued the need for manipulative devices to teach mathematics at the secondary school level and cited the Madison Project's "shoe box kits" as exemplars (p. 118).

So we see that the current desire to use devices to teach mathematics is not new. Mathematics educators have supported their use for 200 years, and the arguments have remained much the same over that period of time. At the same time, it is clear that classroom teachers, particularly those teaching above the primary level, have resisted using these devices. This leads to questions of why they resist and why we should expect teachers to adopt manipulative activities now when they have not done so in the past. I believe the question of resistance centers on two issues. The first is the commonly-held belief that mathematics, at heart, is a pure, abstract, deductive, and solitary endeavor. This belief stems from the Greek tradition, and is enhanced by remnants of "faculty psychology" theory which holds that mathematical thinking helps one to develop powers such as memory and reason. The second factor centers on teachers' perceived responsibility to cover the curriculum to yield the greatest return in student knowledge. To date, those returns have been demonstrated through performance on written achievement tests, and in the past, it has not been convincingly shown that the use of manipulative materials results in enhanced student achievement. Arguments in the past typically were made citing a theory of instruction or appealing to one's common sense.

If we hope to change teaching practice in mathematics, we must address these two issues. To change the public perception regarding the nature of mathematics will require a major exercise in public re-education. This may already be occurring, as evidenced in the popularity of books on fractals and chaos theory as well as those related directly to the nature of mathematics (e.g., Davis & Hersh, 1981). As well, the increasing use of computers and development of interactive software (e.g., *Geometer's Sketchpad,* 1992) should assist in de-emphasizing memory and algorithms as key mathematical skills, and should help to promote the exploratory inductive features of the subject. Finally, the NCTM's unprecedented and massive promotion of the Standards documents is clearly having an effect on the thinking of policy makers, curriculum developers, and textbook publishers. One immediate result has been the increasing availability and visibility of manipulative materials produced by commercial suppliers. Exhibitors' displays at professional meetings include specialized products from small companies as well as a broad array of materials designed to support major textbook series.

At the classroom level, the question of payoff remains. That, too, shows promise of being answered as the results from research into the effectiveness of manipulative materials accumulate. Sowell (1989), for example, conducted a meta-analysis of 60 studies to determine the effectiveness of mathematics instruction with manipulative materials. She concluded that achievement is increased through the long-term use of concrete instructional materials in comparison to symbolic instruction. Treatments of a school year or more yielded moderate to large size effects for elementary grades.

Increasingly, educators are coming to understand that manipulative materials are designed to assist students to develop mathematical understanding rather than to achieve specific mathematical ends. The abacus is used, for example, not as a device to carry out mathematical computations, as it was in pre-Renaissance times, but as a means to foster children's understanding of place value. The almost universal use of the slide rule a generation ago has been replaced by calculators, but the slide rule can serve as a powerful visual and tactile means to understand logarithmic and exponential functions and the associated algorithms for multiplication and division. Such use is similar to accepted pedagogy in science where students may use simple, perhaps primitive, equipment to develop insight into basic scientific principles underlying natural phenomena such as motion, gravity, chemical reactions, and biological processes. The stigma attached to the use of physical devices in mathematics can be reduced by using them regularly and for all students rather than only for "remedial" purposes for students having difficulty in mastering paper-and-pencil algorithms.

Finally, one must consider the place of concrete materials in a world of computers. Computers can generate geometric shapes, simulate probabilistic experiments, calculate arithmetic results, and manipulate symbols in algebraic mode. Computers can provide powerful pedagogical experiences, but it is also the case that they should be used for purposes for which they are best suited. It makes little sense, for example, to focus on computer simulations just to replicate activities that are easily carried out with materials such as algebra tiles, multibase blocks, and attribute blocks. The overdependence on computers, for example, has led to concerns about the qualifications of current graduates of our engineering schools who are becoming divorced from the materials with which they work. The move to computers has eliminated the requirement for students to use their hands, and it is ironic, indeed, that the term "hands-on" was first used to describe a computing situation where the user was able to control the machine by having his or her hands on the keyboard itself. Thus, the increasing role of computers may well force us to recognize the value of manipulatives if we are to avoid becoming more dependent on image rather than substance.

REFERENCES

Bidwell, J. K., & Clason, R. G. (Eds.). (1970). **Readings in the history of mathematics education.** Washington, DC: NCTM.

Bruner, J. S. (1960). **The process of education.** Cambridge: Harvard University Press.

Cohen, P. C. (1982). **A calculating people: The spread of numeracy in early America.** Chicago: University of Chicago Press.

Cooper, B. (1985). **Renegotiating secondary school mathematics: A study of curriculum change & stability.** London: Falmer Press.

Davis, P. J., & Hersh, R. (1981). **The mathematical experience.** Boston: Houghton Mifflin.

Dewey, J. (1938). **Experience and education.** New York: Macmillan.

Dienes, Z. P. (1963). On the learning of mathematics. **Arithmetic Teacher, 10**(3), 115-126.

Fitzgerald, W. M., & Vance, I. E. (1970). Other media and systems. In M. F. Rosskopf (Ed.), **The teaching of secondary school mathematics** (33rd yearbook of the National Council of Teachers of Mathematics). Washington, DC: NCTM.

Geometer's Sketchpad (Version 2). (1992). Berkeley, CA: Key Curriculum Press.

Mathematics Association. (1928). **Report of the Mathematical Association Committee on the teaching of mathematics in public & secondary schools** (Revised ed.). London: G. Bell and Sons.

Piaget, J. (1971). **The psychology of intelligence.** Boston: Routledge and Kegan.

Sowell, E.J. (1989). Effects of manipulative materials on mathematics instruction. **Journal for Research in Mathematics Education, 20**(5), 498-505.

Syer, H. W. (1953). Sensory learning applied to mathematics. In H. F. Fehr (Ed.), **The learning of mathematics: Its theory and practice** (21st yearbook of the National Council of Teachers of Mathematics). Washington, DC: NCTM.

Taylor, E. G. R. (1954). **Mathematical practitioners of Tudor and Stuart England.** Cambridge: Cambridge University Press.

Van Engen, H. (1953). The formation of concepts. In H. F. Fehr (Ed.), **The learning of mathematics: Its theory and practice** (21st yearbook of the National Council of Teachers of Mathematics). Washington, DC: NCTM.